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Heat Transfer Theory – Practical Approach

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1 Introduction

This document was produced and published by Gary Slenders – Principal Process Engineer for AOMC Pty Ltd as part of the education series for Engineering Student in the field of Process, Chemical, and Environmental Engineers as a means of providing a better understanding of the fundamental processes involved in Odour Control. The information contained in this document is based on established material currently in the public domain. You are free to use this material, we ask only that you provide appropriate recognition to the author if you use a significant portion of this document for educational or commercial purposes.

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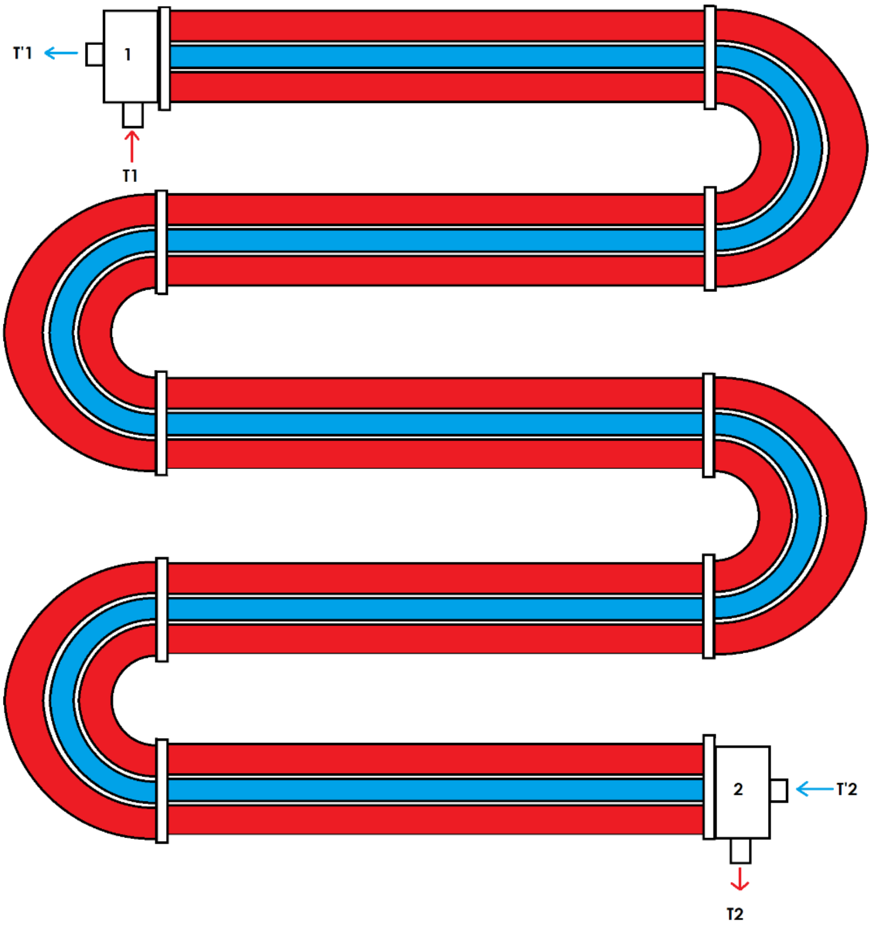
The key purpose of this document is to simplify the understanding of what mass and heat transfer is all about. As such the information presented is not as rigorous presentation as seen in the standard texts. For a more standardised assessment, we always recommend going back to the source documents.

This document is the second part of the series of documentation presented in relation to odour control. In systems, such as BioTrickling Filters and BioFilters it is often necessary to either heat or cool the air stream and heat transfer makes up a major component. Heat management is also an integral part of most Wastewater Treatment Plant operations. Heat recovery from CHP engine and Digester sludge heating are examples of applications for heat transfer.

As previously discussed in Introduction to Mass Transfer, the analogies that exist between the heat and mass transfer mechanisms suggest that it is important to understand both processes. At a later stage, we discuss processes involving both mass and heat transfer in examples such as pack column humidifiers.

2 Simplest Heat Exchanger – Tube in Tube Heat Exchanger

The simplest of heat exchanger designs consists of inner and outer pipe for which one fluid passes through the inner pipe and the other fluid passes through the outer pipe in opposite directions (counter current flow).



$$E = Q_H \rho_H Cp_H (T_1 - T_2) = Q_C \rho_C Cp_C ({T'}_1 - {T'}_2)$$

From basic principles of Mass and Heat Transfer

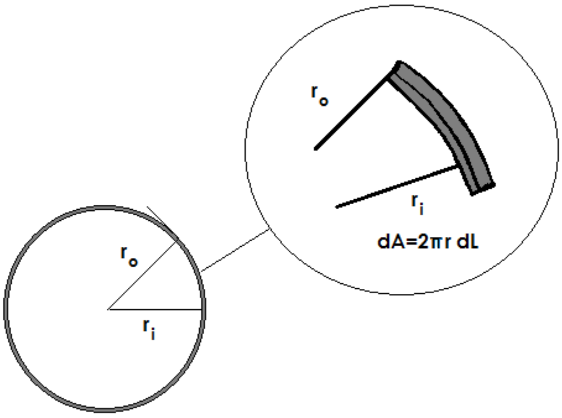
$$dE = Q_H \rho_H Cp_H dT = h_o P (T - T_W) dL = Q_C \rho_C Cp_C dT' = h_i P' ({T'}_W - T') dL$$

$$dE = dA_o h_o (T - T_W) = dA_o h_i \frac{D_i}{D_o}(T'_W - T')\ as\ \frac{A_i}{A_o} = \frac{D_i}{D_o}$$

$$dE = dA\frac{k}{\Delta x}(T_W - T'_W)\ \text{for which}\ \Delta x = r_o - r_i$$

3 Calculating the thermal conductivity through tube

When considering an area element within the inside tube



$$dE = dA\frac{k}{\Delta x}(T_W - T'_W) = 2\pi rk\ dL\frac{dT(r)}{dr}$$

$$\frac{dE}{2\pi k\ dL}\int_{r_i}^{r_o}\frac{dr}{r} = \int_{T'_W}^{T_W} dT(r)$$

$$\frac{dE}{2\pi k\ dL} In\left(\frac{r_o}{r_i}\right) = (T_W - T'_W)\ \text{and letting}\ dA_o = 2\pi k r_o dL = \pi k D_o dL$$

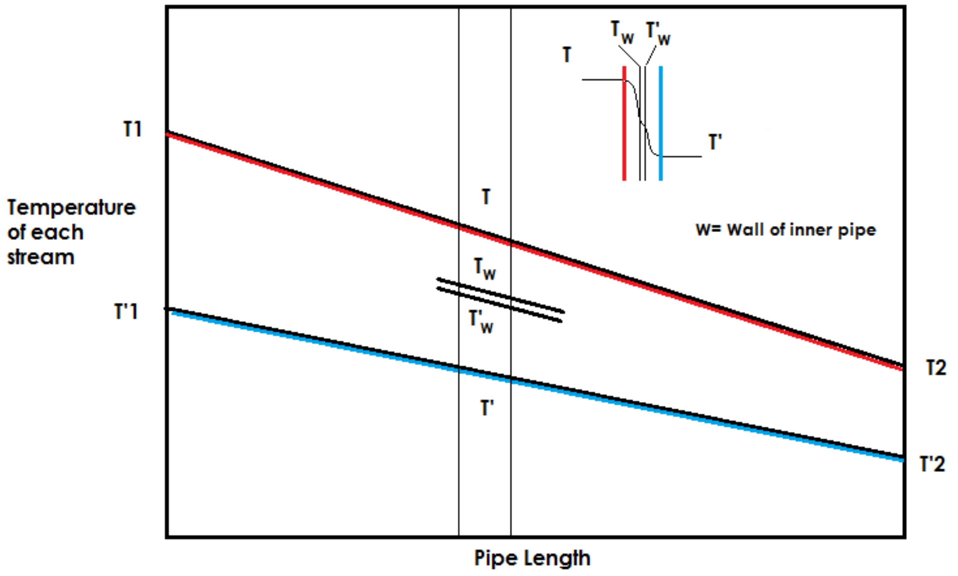
$$dE = 2k\ dA_o\frac{(T_W - T'_W)}{D_o In\left(\frac{D_o}{D_i}\right)}$$

4 Calculating overall heat transfer coefficient

The simple solution to the problem is:

$$\int dE = \int dA_o\, h_o(T - T_W)$$

The wall temperature isn't necessarily easily obtained, but the bulk temperature at each point relative to the cold side and hot side can be obtained



From repeated studies of simple heat exchanger systems (pure counter-current flow) it has been determined that there is a linear gradient between the two sets of end temperatures.

It is possible to consider a single heat transfer value that accounts for the effects of convection on both the inner and outer fluid and the thermal conduction across the wall of the inner pipe.

$$dE = dA_o U(T - T') = dA_o h_o(T - T_W) = dA_o h_i \frac{D_i}{D_o}(T'_W - T') = 2k\, dA_o \frac{(T_W - T'_W)}{D_o In\left(\frac{D_o}{D_i}\right)}$$

$$(T - T') = (T - T_W) + (T_W - T'_W) + (T'_W - T')$$

$$\frac{dE}{dA_o U} = \frac{dE}{dA_o h_o} + \frac{dE D_o In\left(\frac{D_o}{D_i}\right)}{dA_o 2k} + \frac{dE D_o}{dA_o h_i D_i}$$

$$\frac{1}{U} = \frac{1}{h_o} + \frac{D_o In\left(\frac{D_o}{D_i}\right)}{2k} + \frac{D_o}{h_i D_i}$$

This formula holds true only if the transport and thermal properties are consistent over the distance between the outer fluid and the outer surface of the inner tube and the inner fluid and the surface of the inner tube.

Note that temperature variability has not been highlighted at this stage. Specific heat, thermal conductivity, density, and viscosity are temperature dependent and needs to be taken into account in the calculations

Final consideration is the effects of Fouling. To cater for this the Fouling Factor is introduced. The factors are readily available from published sources and should be applied to both the internal and external fluid.

$$\frac{1}{U_D} = \frac{1}{U} + R_F$$

5 Heat transfer across the entire length

If we consider the entire length of the tube in tube heat exchanger:

$$dE = Q_H \rho_H Cp_H dT = -dA_o U(T - T') = -dA_o U(\alpha + \beta T)$$

Where

$$U(T - T') = (\alpha + \beta T), U_1(T_1 - T'_1) = (\alpha + \beta T_1), \&\ U_2(T_2 - T'_2) = (\alpha + \beta T_2)$$

It can be stated in this fashion as U, T and T’ vary linearly to L, and therefore U, T’ must vary linearly for T’

$$Q_H \rho_H Cp_H \int_{T_1}^{T_2} \frac{dT}{(\alpha + \beta T)} = \int_0^{A_o} -dA_o$$

Integrating over the full length

$$\frac{Q_H \rho_H Cp_H}{\beta} ln \left| \frac{(\alpha + \beta T_2)}{(\alpha + \beta T_1)} \right| = -A_o$$

To find β

$$U_1(T_1 - T'_1) - U_2(T_2 - T'_2) = (\alpha + \beta T_1) - (\alpha + \beta T_2)$$
$$\beta(T_1 - T_2) = U_1(T_1 - T'_1) - U_2(T_2 - T'_2)$$

$$\frac{1}{\beta} = \frac{(T_1 - T_2)}{U_1(T_1 - T'_1) - U_2(T_2 - T'_2)}$$

$$E = Q_H \rho_H Cp_H (T_1 - T_2) = A_o \frac{U_1(T_1 - T'_1) - U_2(T_2 - T'_2)}{ln\left|\frac{U_1(T_1 - T'_1)}{U_2(T_2 - T'_2)}\right|}$$

This being the classic equation for heat transfer across two fluids in pure counter-current conditions E = UA LMTD.

6 Evaluating Overall Heat Transfer Coefficient

When considering heat transfer for sections that include smooth pipe assuming formula adopted is by Siedler-Tate

for 50 <N_{RE} < 2100 – laminar flow conditions

$$\frac{hd_e}{k} = 1.86\left[\left(\frac{d_e v\rho}{\mu}\right)\left(\frac{C_P\mu}{k}\right)\left(\frac{d}{L}\right)\right]^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14} \text{ where } N_{Re} = \frac{d_e v\rho}{\mu}$$

N_{RE} > 2500 – turbulent flow conditions

$$\frac{hd_e}{k} = 0.027\left(\frac{d_e v\rho}{\mu}\right)^{0.8}\left(\frac{C_P\mu}{k}\right)^{0.333}\left(\frac{\mu}{\mu_w}\right)^{0.14} \text{ where } N_{Re} = \frac{d_e v\rho}{\mu}$$

The factor $\Phi = \left(\frac{\mu}{\mu_w}\right)^{0.14}$ which accounts for the differences between the wall and bulk stream viscosity is introduced as a means of providing a correction to ensure that linearity of the overall heat transfer coefficient across the profile of the inner pipe and along the length of pipework. The assumed linearity of U assumed that transport properties are linear in respect to temperature. Most are, but viscosity certainly is not.

7 Determining Wall Temperature effects

This key requirement to determining the overall heat transfer by accounting for the viscosity of the fluid at the wall surface requires the temperature to be determined. To calculate this, it is necessary to determine the temperature at either side of the inner tube wall. From earlier determination:

$$dE = dA_o U(T - T') = dA_o h_o (T - T_W) = dA_o h_i \frac{D_i}{D_o} (T'_W - T')$$

$$U(T - T') = h_o (T - T_W) = h_i \frac{D_i}{D_o} (T'_W - T')$$

$$T_W = \frac{T(U - h_o) - UT'}{h_o} \quad and \quad T'_W = \frac{T h_i D_i - U D_o (T - T')}{h_i D_i}$$

Clearly as U, h_o and h_i are functions of wall temperature, this process is iterative with a starting point of assuming the viscosity factor $\Phi = 1$ and repeating until the values converge.

8 Application to real world situations

There is sufficient information to allow a full useable heat exchanger to be designed based on the presented information. There are some aspects to consider when applying this information to other forms of Heat Exchangers:

1. When considering Shell and Tube heat exchangers, designs require inclusion of tube and shell side, pitch, baffle placement, and the degree of co-current and counter current flow. Shell side heat transfer coefficient is given in different correlations
2. Plate and Spiral Heat exchangers have a very high degree of counter current flow and again have their own correlations for determining heat transfer coefficient.
3. Pressure Drop calculations need to be taken into account and the operating cost to the system. There are already extensive formulations available on determining pressure drop in each specific device.
4. Most significant issue in any heat exchanger design is to determining the transport and thermal properties for each fluid for the inlet and outlet conditions.

9 Variables and Units

Variable	Units	Description
a_w	m^2/m^3	Wetted Specific Surface Areas
C_P	J/kg C	Specific Heat of Fluid
d_e	m	Pipe Diameter, Equivalent Diameter
E	W (watts)	Power gain or loss by way of heat transfer
f	-	Fanning's friction factor
g	m/s^2	Acceleration due to gravity
h	W/m^2 C	Heat Transfer Coefficient
k	W/m C	Thermal Conductivity of fluid or solid
L	m	Length of pipe
P	m	Perimeter length of pipe
r	m	Radius of pipe
R_F	m^2 C/ W	Fouling Factor
Q	m^3/s	Volumetric Flow
v	m/s	Bulk flow velocity
V	m^3	Volume
T, T'	C	Temperature
ρ	kg/m^3	Bulk Fluid Density
μ μ_W	kg/ms	Bulk fluid viscosity, wall fluid viscosity

10 Literature Cited in this Document

- Perry’s Chemical Engineer’s Handbook 7th Ed – R H Perry and W D Green
- Principles of unit operations 2nd Ed - A. Foust et al
- Process Heat Transfer – Q D Kern
- Alfa-Laval manual of design